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ABSTRACT

Nineteen research reports related to mathematics education are abstracted and analyzed. The reports abstracted were selected from 4 educational journals, 2 psychological journals, and a mathematics journal, and from technical reports issued by the University of Wisconsin's Research and Development Center. The research articles deal with instructional materials (manipulatives, calculators), student learning of mathematics topics (measurement, concept learning, problem solving, geometry and spatial representation, distributive law), other instructional concerns (remedial instruction, feedback, advance organizers), and general educational concerns (school entry age, sex differences in mathematics achievement, textbook adoption, test development). Research related to mathematics education which was reported in RIE (Resources in Education) and CIJE (Current Index to Journals in Education) between July and September 1975 is listed. (DT)

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Vol. 8, No. 4 Autumn 1975

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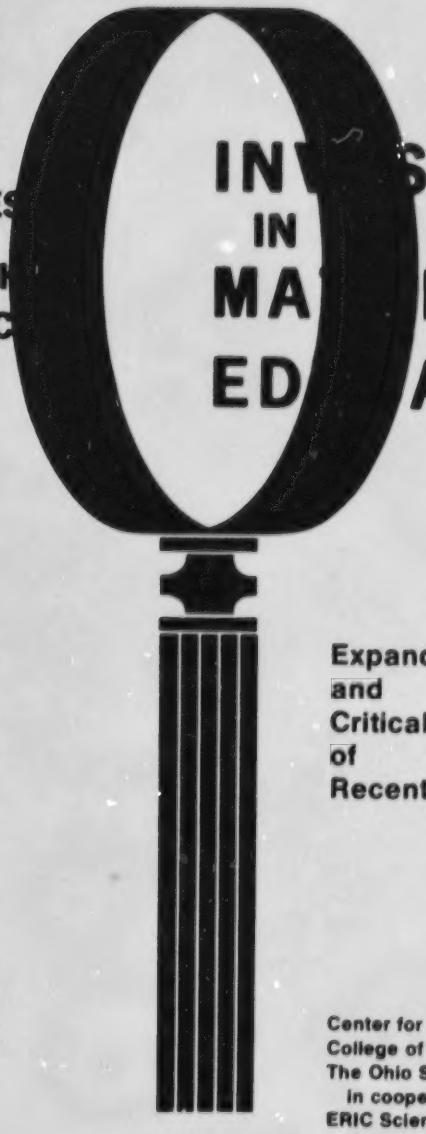

INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts
and
Critical Analyses
of
Recent Research

Center for Science and Mathematics Education,
College of Education,
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in cooperation with the
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SE 021 550

INVESTIGATIONS IN MATHEMATICS EDUCATION

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COGNITIVE RESULTS BASED ON DIFFERENT AGES OF ENTRY TO SCHOOL: A COMPARATIVE STUDY. Austin, Gilbert R.; Postlethwaite, T. Neville. Journal of Educational Psychology, v66, pp857-863, December 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James E. Bierden, Rhode Island College.

1. Purpose

To look for a relationship between age of entry into school and achievement in mathematics, science, and reading comprehension, as mensured at ages 10 and 13.

2. Rationale

Within the last 15 years, much has been written about the importance of early childhood education as a basis for later school learning. Important contributions to this literature have led to the conception and development of such programs in the United States as "Head Start" and "Follow Through". American efforts have been duplicated in a number of other countries. Many of these projects have made the assumption that early entry into school would have a beneficial effect on the children's later school achievement.

In a number of national studies, the benefits of these early school intervention programs have been very difficult to document, primarily because very little variation in effective age of entry into school occurs within a country.

An alternate method of looking at this question is to examine the results of multinational studies. The primary goal of these studies was not to answer questions about age of entry into school. However, the assumption is made that data banks from these studies can be used in secondary analyses to give some insights about the problem.

3. Research Design and Procedure

The study took data from three International Educational Achievement (IEA) studies in mathematics, science, and reading comprehension. These IEA studies are cross-sectional surveys with small standard errors of sampling. Data used were national mean scores for 13-year-olds for mathematics, 10-year-olds for science, and 10-year-olds and 14-year-olds for reading comprehension. The number of observations (national mean scores) was limited, typically to 12 or 13.

These data were reanalyzed in relationship to both the official age of entry into school and effective age of entry obtained from Organization for Economic Cooperation and Development studies. Effective age of entry was defined as the year in which 75 percent or more of an age group enter formal school.

Tables and scattergrams were produced showing the relationship between mean scores and both official age of entry and effective age of entry.

4. Findings

The findings of the relationships between mean scores and ages of entry are summarized in Table 1 using r statistics reported for each scattergram.

Table 1

r - Coefficients for Entry Age vs. Mean Scores

Score	Official Entry Age	Effective Entry Age
Mathematics		
Population 1A (Age 13)	-0.23	-0.37
Population 1B (Age 13)	-0.38	-0.49
Reading Comprehension		
Age 10	0.22	0.43
Age 14	0.04	0.03
Science		
Age 10	0.42	0.39

As seen from the table, earlier age of entry is associated with higher scores in mathematics and with lower scores in reading and science at age 10.

5. Interpretations

The design of this study forces speculation rather than interpretation. Nevertheless, the authors present possible explanations for the contradictory results between reading and mathematics and for the scores in science.

a. Based on the developmental sequencing theories of Piaget and Montessori, it is possible that preschool sensorimotor activities transfer to beginning mathematics skills more easily than they do to reading comprehension skills.

b. Qualitative planning for the presentation of materials to students has been found to be necessary in early childhood education. Mathematics is generally judged to be more easily sequenced than reading in this regard.

c. High correlations between mathematics scores and ratings of "opportunity to learn" (actual, as opposed to intended, curriculum) found in other IEA studies are used to argue that degree of emphasis does make a real difference in mathematics achievement.

d. The quality of homes from which children come may make a difference. In many countries reading is not given much attention beyond the mechanics. Thus, it is surmised, reading comprehension redounds to the home and is then much more associated with variations between homes than between schools.

e. The home may play a part in science learning, too. According to other IEA data, science is taught spasmodically up to the age of 10 in most schools. Thus it may not be surprising that the longer children remain at home the higher their achievement on IEA science tests.

Critical Commentary

The implications of early childhood education extend far beyond the schools. Social, political and even financial considerations are part of the discussions about the importance of early entry into school. Because of these larger ramifications, great care should be taken in the design of studies dealing with this topic. This care is obviously lacking in the study reported here. The authors readily admit as much. Any interpretation of the "effect" of age of entry must be speculative, given the lack of control of other factors.

The problems associated with control of these factors is formidable to say the least. A theoretically sound research design would assign students at random to various groups, have each group start school at a different age but be subjected to identical schooling, and then test the students' achievement at some uniform age. Such experimental research would be impossible to conduct in practice within any educational system. The problems of how to control for variability of factors both within countries and between countries are acknowledged but not addressed in this study. This increases the speculative nature of the investigation.

On the other hand, early childhood education is important enough to make even speculative analysis desirable. In this case, secondary analysis of primary data becomes a useful research methodology. Constraints of time and finances usually mean that only a fraction of the possible analyses have been undertaken in large data collection projects. In using these data banks for secondary analysis, researchers will always face the problem that the data were created for purposes other than for the new focus. These problems are readily apparent in the present study. However, properly executed secondary analyses of primary data can make valid and valuable contributions. The present study makes such a contribution and suggests methodologies for subsequent uses of this research tool.

James E. Bierden
Rhode Island College

EFFECTS OF MANIPULATIVE ACTIVITIES ON ARITHMETIC ACHIEVEMENT AND RETENTION. Bledsole, Joseph C.; Purser, Jerry D.; Frantz, Nevin R., Jr. Psychological Reports, v35, pp247-252, 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leland F. Webb, California State College, Bakersfield, California.

1. Purpose

The study was designed to determine whether:

a. For all ability levels and for both boys and girls, the experimental method utilizing manipulative activities will produce greater achievement and retention than the conventional method.

b. Differences would be obtained among three ability levels in the order of high greater than middle greater than low, for both achievement and retention tests.

c. No sex differences would be found.

d. No significant first- or second-order interactions (method, ability, and/or sex) would be found.

2. Rationale

Since various proponents of career education advocate correlating vocational and academic programs, it is hypothesized that utilizing manipulative objects (in this case, a steel scale and micrometer to measure objects) would enhance mathematical learning and retention. Based upon a literature review in the area of the use of manipulatives, it was further hypothesized that there were several different relationships requiring investigation in this study: (1) the instructional treatment, (2) the ability level of the subject, and (3) the sex of the subject.

3. Research Design and Procedure

The subjects were 339 seventh-grade students, of all economic and social levels, selected from a southeastern town of 50,000 people. Fifteen intact mathematics classes comprised the population. Previously, students were assigned by the school system by ability to a specific class. From the fifteen classes, twelve classes were randomly selected for the study. From each ability group (high, medium, low), two classes were assigned to each treatment. The sample size for the experimental group was 169 and for the control group, 170.

The experimental treatment was a series of learning packages dealing with decimals and fractions, followed by manipulative activities using a scale and micrometer to measure objects. The control treatment included the same series of learning packages, and, in lieu of manipulative activities, a second series of learning packages on measurement with a scale and micrometer was utilized.

Teachers were given instructions on how to administer the learning packages and units. The Basic Skills in Arithmetic Test was administered as a pretest on the day before the experiment commenced. The test, developed in 1954, measures command of 43 fundamental skills in mathematics which involve whole numbers, decimals, fractions, and percentages. The learning packages and criterion test were previously pilot-tested on 67 students at a separate junior high school, indicating the criterion test was appropriate for all ability levels.

The length of the instructional treatment varied between four and eight days. On the average, high-ability students completed the learning packages in four days, while the medium- and low-ability groups took about six and eight days, respectively. The posttest and the retention test were the same as the pretest (the Basic Skills in Arithmetic Test), and were administered to each group upon completion of the learning package and one month later. Since different ability groups completed the package at different times, these tests were not administered all at the same time. The pretest was utilized as a covariate in a treatment by levels by sex analysis of covariance design. The dependent variables were the criterion test and the retention test.

4. Findings

The analysis of covariance indicated significant differences on both criterion tests in favor of the experimental treatment ($F_{1,327} = 27.39$, $p < .001$, for the posttest; $F_{1,327} = 63.50$, $p < .001$, for the retention test). The hypothesis that students of high ability would make greater gains than students of medium and low ability was also confirmed for the posttest ($F_{2,327} = 10.16$, $p < .01$), but not for the retention test ($F_{2,327} = 0.67$, $p > .05$). No sex differences were found on the posttest, but there was a significant sex difference favoring girls on the retention test ($F_{1,327} = 4.50$, $p < .05$). No significant first- or second-order interactions between treatment, ability or sex were found.

5. Interpretations

With respect to the high-, medium-, and low-ability groups, the adjusted means in the analysis of covariance design reduced the retention test results essentially to random differences, although the actual means are higher. This is a remarkable finding, in that on the posttest significant differences were found, but they did not carry over to the retention test results. The fact that students of low ability performed adequately enough on the retention test to result in no significant difference in ability level is noteworthy.

Results of the exploratory study suggest that manipulative activities may be an effective means of producing both retention and achievement of number operations with fractions and decimals. Students of low ability were able to overcome significant differences on the posttest favoring higher ability levels, an indication that students performing higher on the posttest forgot more than the low-ability students by the time the retention test was taken.

The study suggests that seventh-grade students taught with learning packages that include manipulative activities achieve and retain significantly more on a criterion test involving fractions than students taught by a conventional method.

Critical Commentary

While the study itself is a reasonable one to undertake, and the use of the statistics is appropriate, there are several questions which need to be raised regarding some of the statements made in the article:

1. The authors employ an analysis of covariance design and indicate they are primarily concerned with manipulative versus non-manipulative activities. The literature review is restricted to a discussion on the use of manipulatives and related studies, but no mention is made of studies related to previous findings with respect to sex differences or ability level differences.

2. While the literature search suggests that findings have been conclusive regarding the use of manipulatives, the researchers use a directional hypothesis in their study, which seems to be unfounded, based on the previous findings in this area.

3. The sample sizes of each group and the adjusted means are reported in various tables, and F-ratios and significance levels are reported in the body of the study. It would have been helpful to have a table with means and standard deviations reported, along with a complete source table reporting mean squares, sums of squares, and all F-ratios. It is noted that when reporting the F-ratios, the authors used significance levels of .05, .01, and .001.

4. The use of the same ability test as a covariate and then as the criterion test and the retention test may be a cause of concern. The pretest was administered one day before the learning packages were distributed, and students of high ability took the same test five days later. Students of medium and low ability took the test again seven and nine days later. This appears to be a very close interval between the administration of the same test, one which might account for the significant differences on the ability levels. It could be just as easily conjectured that students in the high-ability group achieved higher scores because they took the posttest five days later, instead of seven or nine days later. This could also account for the lack of significant differences between the ability levels on the retention test, because this difference would diminish over time. It might have been more appropriate to have selected another standardized test for use as a covariate.

5. The posttest was administered to each group when they completed the unit, but the authors do not indicate what was done if a student in a particular group finished early. If a student of low ability finished in four days, what did he do during the additional four days before the posttest was administered?

6. While the authors state that the experimental and control treatments are designed to teach fractions and decimals, the test selected for

the covariate, criterion, and retention test encompasses whole numbers, fractions, decimals, and percentage. The use of a subscore on the test involving fractions and decimals might have been more appropriate.

7. There is an indication in the article that the high-ability group made greater gains than students in the middle-ability group than students in the low-ability group. It is not clear how this was determined. An analysis of covariance test will indicate whether there are significant differences between more than two groups, but it will not indicate precisely between which groups the significance exists. The authors imply significant differences between all three groups, but no indication is given of any additional statistical test (e.g., Duncan's New Multiple Range Test) having been conducted to confirm this conclusion.

This study has some long-range significance, but there are a number of concerns cited above which tend to negate the study in question. While the statistical analysis is appropriate, the questions dealing with the relationship between the hypothesis and literature search, the selection and use of the criterion and retention test, and other factors as indicated, create some cause for concern. A modification and division of this study into smaller components for the purposes of replication would be an appropriate approach to take for further research in this area.

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SE 015 930

THE DEVELOPMENT OF THE CONCEPT OF A STANDARD UNIT OF MEASURE IN YOUNG CHILDREN. Carpenter, Thomas P. and Lewis, Ruth. Journal for Research in Mathematics Education, v7 n1, pp53-58, January 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward C. Beardslee, Seattle Pacific College.

1. Purpose

To determine whether young children are able to predict that there will be an inverse relationship between unit size and number of units when measuring a fixed quantity before they recognize that they may not compare quantities measured with different units.

2. Rationale

It has been found that when children were asked to measure equal quantities with different-sized units of measure, more of them maintained that the quantity that measured more units was greater, in spite of the fact that before measuring they had visually determined that the quantities were equal (Carpenter, 1971, 1975; Gal'perin and Georgiev, 1969; Piaget, Inhelder, and Szeminska, 1960).

Also, in certain conservation studies (Flavell, 1963; Halford, 1969), it has been noted that children recognize the compensating relationship between the width of a container and the height of the liquid in the container before they are able to conserve. Hence it is hypothesized that children who ignore differences in unit size in making measurement comparisons recognize the effect of changing unit size and that there is a compensating relationship between unit size and number of units.

3. Research Design and Procedure

Four measurement problems were individually administered to a group of 51 first- and second-grade children over a three-day period in January 1974. The problems consisted of two types of measurement, length and capacity, and two types of tasks, prediction and comparison.

For example, the length prediction item consisted of showing each subject two strips of paper that were the same length. After the subject verified that the two pieces were the same length, the two strips were placed in a T-configuration to prevent further direct comparison. Then the experimenter placed four boxes (the larger unit) along the horizontal strip, and the subject was asked how many of the smaller boxes could be placed along the other strip. (The ratio between the length of the boxes was seven to five.) Any response of more than four boxes was considered correct.

The linear comparison problem was similar to the prediction problem except that after the strips had been visually compared and placed in a T-configuration, five of the larger boxes were placed along the horizontal

strip and seven of the smaller boxes were placed along the other strip. Then the subjects were again asked to compare the lengths of the strips.

The capacity problems were similarly designed using an opaque container and two measuring units. The main distinction between the length and capacity tasks was that after the liquid was measured into an opcque container, the subject could not compare the quantities; there was no visual reinforcement after measuring as in the case of the length problems.

Subjects were randomly assigned to two groups. One group was administered linear items first and the other group was administered liquid items first. In each case, prediction items preceded the comparison items and in every item the larger unit was used first (with the smaller unit used as the predicting unit in the prediction items).

All items were administered individually in a small room set apart from the classrooms, by an experimenter who was acquainted with the majority of the children. Each subject sat across the table from the experimenter. The experimenter manipulated quantities and units. Procedures and protocols were kept as consistent as possible for all items.

4. Findings

Thirty-one subjects correctly answered the capacity prediction problems and 35 subjects correctly answered the length prediction problems, while only 5 subjects correctly answered the length comparison problems and 5 subjects correctly answered the capacity comparison problems. The results were analyzed using the 2x2 repeated measure ANOVA. Prediction problems were significantly easier than the comparison problems, but there was no significant difference in difficulty between the length and the capacity problems.

An analysis of errors indicated that errors in the prediction problems resulted from the subjects perceiving a direct rather than an inverse relation between unit size and number of units. Errors on the comparison problems resulted in selecting the quantity that was measured with the greatest number of units or by simply choosing the quantity that was measured with the larger unit.

5. Interpretations

Results of this study support the results of earlier studies. Further, the unit-size/number-of-units compensating relationship seems to develop independently of experience with different units and parallels the development of compensation in the traditional conservation problems. This finding tends to support Piaget's conclusion that there exist basic cognitive structures that, once developed, can be applied to various similar behavioral situations.

The results of this study appear to indicate that manipulation with different units of measure do not contribute to an understanding of the unit-size/number-of-units relationship and may tend to reinforce incorrect notions of quantity. Thus, it could be hypothesized that concrete

materials may not aid in the development of certain concepts and may in fact interfere with their acquisition or obscure their underlying principles.

Critical Commentary

The suggestion that the use of manipulative materials may not facilitate the acquisition of concepts is questionable on the basis of this study, since only four particular items were involved. Also, the subjects did not personally manipulate the material. In addition the report of the study suggests that for the length problems thin flexible materials (pieces of paper) were used and that the measuring units were square. Would the same results be obtained if rigid materials (e.g., wooden rods) and rectangular measuring units were used? Another concern in the study is the effect of the ratio of the length of the larger box to the smaller box which was 7 to 5. Would similar results hold if the ratio of the longer unit to the smaller unit was 9:8, 2:1, 3:1? The unit-size/number-of-units relationship could be a function of the unit size. Further studies which vary the unit size and the length of the original object should be undertaken. Also the conclusion that the subject's ability to predict is better than the ability to compare should be tested in other prediction settings: e.g., given only one object to measure, and two different-sized units, can the subject measure with one unit and predict the number of the other units needed? Or without measuring, could the subject predict of which unit would more be required to measure the object? Or in a more abstract setting, if five of one unit measured the length (showing the object measured by 5 units) and telling the subject that 7 of another unit measured the same object, would the new unit be shorter or longer? These suggested variations could help to determine to what extent the desired relationship can be predicted.

One of the significant contributions of this study in addition to supporting previous results is that it is a useful step in developing a standard experiment which can be used to identify whether a child understands the unit-size/number-of-units relationship.

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SE 016 244

THE RELATIVE EFFECTIVENESS OF FOUR STRATEGIES FOR TEACHING ALGEBRAIC AND GEOMETRIC DISJUNCTIVE CONCEPTS AND FOR TEACHING INCLUSIVE AND EXCLUSIVE DISJUNCTIVE CONCEPTS. Dossey, John A. Journal for Research in Mathematics Education, v7 n2, pp92-105, March 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James K. Bidwell, Central Michigan University.

1. Purpose

The study investigated the role played by teaching strategies and exemplification approaches in the attainment of certain algebraic and geometric concepts and of inclusive and exclusive disjunctive logic concepts.

2. Rationale

In a previous paper (see I.M.E., v7, pp17-20, Winter 1974), reporting analysis of the same data, significant differences were found between Ss' attainment of the concepts mentioned above. This present report reanalyzed data for the effect on strategy and exemplification approaches.

3. Research Design and Procedure.

A 4x2x2 completely crossed factorial design was used which consisted of 4 strategies, 2 exemplification modes, and 2 intelligence levels. The four strategies were (see Henderson, 1970, for details):

CE: 4 characterization moves, then 6 exemplification moves

CEC: 2 characterization moves, then 6 exemplification moves, then 2 more characterization moves

ECE: 3 exemplification moves, then 4 characterization moves, then 3 exemplification moves

EC: 6 exemplification moves, then 4 characterization moves

The two exemplification modes were:

Example approach (E) - 4 example moves and 2 nonexample moves used in each strategy.

Nonexample approach (~E) - 6 example moves and 4 nonexample moves used in each strategy

The two intelligence levels were high (H) and low (L), based on the Hermon-Nelson Test of Mental Maturity.

The Ss consisted of 320 prospective elementary school teachers selected from 363 students enrolled in an undergraduate mathematics course.

The Ss were ranked 1 to 363 on the Hermon-Nelson test. The top 160 were assigned to the H group and the low 160 to the L group. Within these groups random assignment was made to the remaining cells of the design.

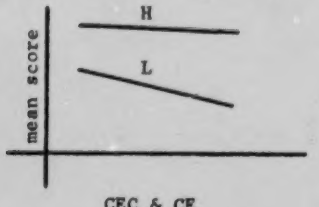
Twelve disjunctive concepts were incorporated into eight programmed instruction units, one for each factor of the design. The concepts were "balanced" with respect to algebraic and geometric content and exclusive and inclusive disjunctive forms. One algebraic exclusive concept was a minp: an ordered pair of natural numbers, (x,y), such that either x = y or x < y. One geometric inclusive concept was an absect: two triangles which have the same area or same perimeter. The Ss completed the programs entirely in class.

A 72-item test, with 6 items for each concept, was constructed to test Ss' acquisition of the concept. The KR-21 reliability was 0.85. The test was administered five days after completion of the program.

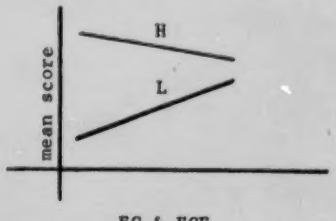
4. Findings

A three-way analysis of variance showed that the strategy means were significant at the 0.05 level for algebraic items and for the inclusive disjunctive items. The same analysis showed that the exemplification means were significant at the 0.01 level for the inclusive disjunctive items. The ability means were significant at the 0.01 level on all analyses. There was a significant 3-way interaction between the main factors on the geometric items and the exclusive disjunctive items.

Further analysis showed that the CEC strategy was superior to the ECE for the algebraic items and the inclusive disjunctive items at the 0.05 level. Analysis of the 3-way interaction yielded the following unexpected trends on means:



CEC & CE



EC & ECE

That is, ~E exemplification produced a higher mean than did E for the EC and ECE strategies in the L ability group.

5. Interpretations

a. The findings emphasize that the logical and subject-matter nature of the concepts has effects on instructional strategy.

b. The strategy factor was significant for algebraic and inclusive disjunctive concepts - areas identified as being harder to attain than their counterparts. Here the CEC strategy was more effective than the ECE strategy.

c. The CEC strategy may be superior to the ECE because of the role and order of the moves. The CEC begins and ends with characterization moves, whereas the ECE begins and ends with examples that require inference to obtain the concept.

d. "A study needs to be conducted with varied numbers of moves... or with evaluation after a certain number of moves."

e. The inclusive disjunctive concepts were superior where more example than nonexample moves were used. This may be caused by the requirement that an example must have one or both of the defining attributes, whereas a nonexample shows none of the attributes.

f. No logical reason can be found for the results of the three-way interaction analysis.

Critical Commentary

Since this study used experimenter-written material and tests, more details of this content would be helpful. No indication of time of study on the material was noted. Were all Ss subjected to the same study time or was the test given five days after the last students were finished? Resulting data may be affected by this factor. How were the nonexamples distributed in the ECE strategy? Initial contact with one or two nonexamples may have been deleterious. Why were non-characterization moves not considered? In the E concepts 8 moves were positive, while only 2 were negative; in the ~E concepts 6 moves were positive and 4 were negative. Nonexamples are conjunctive in nature. If conjunctive logic is easier than disjunctive, this explains the success of the L group in the ~E concepts, if the strategy began with nonexamples. Since the Ss were prospective elementary teachers, had they been exposed prior to this experiment to inclusive "or" in the logic part of the course? This may have affected the test scores.

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EJ 093 533 SE 010 732

SOLUTION METHODS USED IN SOLVING ADDITION AND SUBTRACTION OPEN SENTENCES. Grouws, Douglas A. Arithmetic Teacher, v21, pp255-261, March 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeanette Summerfield, Novato, California.

1. Purpose

This study was designed: (1) to identify the methods used by third-grade children in solving certain open sentences and determine their frequency of use; (2) to ascertain the extent to which different methods were used in solving similar sentences; and (3) to examine the relationship between the number of correct solutions and the number and kinds of solution methods employed.

3. Rationale

Studies involving children's ability to solve simple addition and subtraction sentences at different grade levels, and factors related to differences in difficulty between sentences of various types, have been reported in several journals. It is also important to know what methods children use in solving open sentences. Knowing these methods would be a valuable asset for any teacher in designing and implementing an instructional approach.

3. Research Design and Procedure

Sixteen open sentences, four of each of the forms $N + a = b$, $a + N = b$, $a - N = b$ and $N - a = b$, where N was a placeholder for a number and a and b were whole numbers, were used. Each type included two basic-fact combinations and two two-digit numbers which required regrouping when they were added or subtracted to find the solution. Sixteen boys and sixteen girls were randomly selected from three elementary schools and individually tested. The test was orally administered in two parts on two different days. The same interviewer tested all children in a room apart from the regular classroom.

The interviewer made sure the student could read the sentence correctly, then asked for the solution. Students could use paper and pencil if they wished. The children were also asked to explain their method of solution.

4. Findings

The study identified many methods that third-grade children used in solving open sentences. These are described in Table 1. The frequency of use of solutions is summarized in Tables 2 and 3.

Table 1

Descriptions of Method Classifications

Method	Description of the Method
Meaningful recall	The child's response* is usually correct and is given immediately. Also, evidence indicating comprehension of the number sentence involved is obtained by subsequent questioning by the interviewer.
Rote Recall	The child's response is given immediately but no evidence of comprehension of the number sentence involved is obtained by the interviewer's subsequent questioning.
Simplifying	The child, in determining his response, considers a sentence of the same type as the given sentence only it involves different numbers, usually smaller numbers.
Inverse relationship	The child uses the inverse relationship between the operations of addition and subtraction ("doing-undoing") in determining his response.
Tallying	The child, in determining his response, uses a tally system, usually making some type of marks on the paper provided.
Operation addition	The child uses a traditional addition algorithm on the paper provided in arriving at his response.
Operation subtraction	The child uses a traditional subtraction algorithm on the paper provided in arriving at his response.
Counting on	The child uses a forward counting procedure in arriving at his response.
Counting back	The child uses a backward counting procedure in arriving at his response.
Systematic substitution	The child uses a substitution procedure and a systematic correction procedure in determining his response.
Random substitution	The child uses a substitution procedure, but does not appear to use a systematic correction procedure in determining his response.
Equivalent canonical sentence	The child writes and uses a canonical sentence equivalent to the given sentence in determining his response.
Equivalent noncanonical sentence	The child uses a noncanonical sentence equivalent to the given sentence in determining his response.
Addend-sum relationship	The child determines his response one digit at a time using the operation in the given sentence and the digits of the numbers in the given sentence, or writes the given sentence in vertical form prior to proceeding as above.
Guessing	The child's response, usually incorrect, is often given promptly. Also, the child is frequently unable to indicate his method of solucion or to give a justification for the correctness of his response.

No attempt	The child makes no attempt.
Indeterminate	The child responds, but the interviewer is not able to determine the method used to obtain the response, although evidence indicating the existence of a method is obtained.
Miscellaneous	The child's solution method does not fit any of the previously described categories.

*The phrase "the child's response" refers to the child's numerical response to the interviewer's request to state the solution set of the open sentence under consideration.

Table 2

Frequency of Use of Solution Methods

Solution Method	Subclassification	Frequency of Use
Operation	Addition (52)	196
	Subtraction (144)	
Recall	Rote (18)	67
	Meaningful (49)	
Counting	Counting on (47)	57
	Counting back (10)	
Substitution	Systematic (31)	38
	Random (7)	
Addend-sum relationship		28
Inverse relationship		22
Undetermined		24
Miscellaneous		18
Tallying		15
Simplifying		14
Guessing		11
No attempt		11
Equivalent sentence	Canonical (0)	11
	Noncanonical (11)	

Note: The numbers in parentheses indicate the frequency of use of methods which were combined under a more inclusive general heading.

Table 3

Frequency of Use of Solution Methods by Number Size

Solution Method	Frequency of Use	
	Basic-fact Situation	Two-digit Situation
Recall	67	0
Operation	60	136
Addend-sum relationship	2	26
Counting	32	25
Substitution	21	17
Inverse relationship	24	0
Tallying	8	7
Simplifying	7	7
Equivalent sentence	9	2
Undetermined	9	13
No attempt	0	11
Guessing	6	5
Miscellaneous	11	7

5. Interpretations

The method used most frequently was the operation category that involved the use of the traditional algorithm for finding a sum or difference of two whole numbers. The investigator conjectured that part of the reason for this is associated with the age of the children and the type and amount of formal instruction they have had on solving open sentences. He suggests that more specific reasons must await observational data on children's prior mathematical years.

Another important finding is the considerable number of informal methods of solutions used, indicating that not all children seem to be operating at the same level of maturity. A substantial correlation (.48) was found between the solving performance of children and the number of different solution methods used. That is, children using fewer methods tended to solve correctly more open sentences. Also, good performers used the formal algorithm approximately 78% of the time. However, the author stresses that a potential advantage of an approach to solving open sentences based initially on student-generated methods is that children's flexibility of thinking may be enhanced. He further suggests that, at some point in an instruction sequence which encourages informal solution methods, children should be encouraged to begin solving open sentences by using more efficient means.

Critical Commentary

This was an interesting study in that a surprisingly large number of methods of solution of simple open sentences were identified by the investigator. These methods were explained clearly.

The author points out that there is evidence to indicate that the initial instruction associated with solving open sentences which encourages children to find their own solution methods may have important cognitive and affective outcomes. A question for further research might be: Do such children have difficulty making the transition to more formal and efficient methods, or does such preliminary experience actually facilitate the transition?

The author suggests that teachers should be aware of potential solution methods in order to suggest appropriate ones to his students. Although frequency-of-use data were provided, it would be helpful to see some data regarding the percentage of correct solutions attained by the children for each of the methods listed. This would give teachers some idea of the efficiency of each of the methods.

Another question for possible further study is: Is there a hierarchical instructional sequence of informal and formal methods of solving simple open sentences? It may be that students can make the transition more easily if the instructional sequence is so ordered.

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EJ 091 985 SP 502 304

SEX DIFFERENCES IN MATHEMATICS ACHIEVEMENT - A LONGITUDINAL STUDY. Hilton, Thomas L. and Berglund, Gosta W. Journal of Educational Research, v67 n5, pp231-237, January 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Grayson H. Wheatley, Purdue University.

1. Purpose

To test the hypothesis that sex differences in mathematics achievement result from sex-typed interests. It was predicted that

(a) preadolescent differences in mathematics achievement would be negligible, and

(b) beginning with adolescence, differences in achievement would appear and widen in concert with differences in interests between the sexes.

2. Rationale

It is well documented that boys score higher on measures of mathematics achievement at the secondary level (Tyler, 1965; Anastasi, 1958; Maccoby, 1966). Evidence for this statement has come from numerous cross-sectional studies. Several authors (e.g., Maccoby, 1966) have attempted to explain these differences on the basis of changing interests between the sexes. Although other hypotheses have been put forth (Harris, 1973; Vandenberg, 1973), the authors of this study attend only to cultural factors, namely sex-typed interests, as a possible cause for the differential achievement that has been observed. The present study attempted to determine the relationship between mathematics achievement and sex-typed interests over a seven-year period with the same pupils.

3. Research Design and Procedure

A longitudinal design was used to study the changes in mathematics achievement and interests over the years 1961 to 1967 with pupils beginning in the fiftn grade. The data were obtained from the Growth Study begun at Educational Testing Service in 1961. The same students were retested in grades 7, 9, and 11. The pupils were classified as academic or non-academic on the basis of their programs of study. There were 632 boys and 688 girls in the academic group and 249 boys and 290 girls in the non-academic group. Three instruments were employed to assess changes in mathematics achievement and interests. The Sequential Test of Educational Progress - Mathematics (STEP); the School and College Ability Tests, verbal and quantitative (SCAT -V, SCAT -Q); and a 177-item Background and Experience Questionnaire (BEQ) were given to all pupils each year of the study except that the BEQ was not administered in the initial year. The SCAT and STEP tests have been vertically equated and thus were treated as though the same test had been administered each year. Attempts were made to control dropout rate, the amount of mathematics studied, and the curriculum.

4. Findings

As would be expected, the arithmetic means and standard deviations increased with age, without exception. In STEP-Math males and females were equal at grade five but males had successively higher mean scores at subsequent grade levels. The sex differences were greater for the academic group. For SCAT-Q the same pattern emerged but the differences were not significant until grade 11. On the MEQ there were increasing sex differences on interest in mathematics and science with age.

5. Interpretations

The findings of this study clearly indicate a relationship between mathematics achievement and sex-typed interests. At the fifth-grade level boys and girls are equivalent in mathematics achievement, with any observable difference in favor of girls. For interest in mathematics and science, significant differences begin emerging at grade nine with differences widening at the eleventh-grade level. Thus the differences in achievement and interests parallel each other. This becomes markedly clear when the trend lines are plotted for mean differences between sexes in STEP-Math scores and in "perception of mathematics as potentially useful." For each program group (academic and non-academic) the changes occur in concert with each other. The authors conclude that there is a definite relationship between achievement and sex-typed interests, but indicate that no causal relationship can be inferred. However, they proceed to state that "we see no need to hypothesize physiological or psychological explanations for the disparity in mathematical achievement between the sexes."

Critical Commentary

This study is a significant addition to those research studies investigating sex differences. Longitudinal data are usually the best evidence for testing hypotheses. However, the authors of this paper seem to take too narrow a view of the possible explanations of the differences. In the last sentence of the article (quoted above), the authors ask the reader to ignore any competing hypotheses, thus in fact implying the causal relationship they were so careful to point out could not be concluded. Actually, other hypotheses are tenable in light of consistent spatial ability differences and differences in hemispheric lateralization. The data on mathematics achievement differences are mixed (Maccoby and Jacklin, 1974; Fennema, 1974). While the evidence for a cultural explanation of sex differences is strong (undoubtedly cultural differences are factors), other factors must be considered. Vandenberg (1973) argues that spatial ability is more strongly inherited than other abilities. Since spatial ability is one trait on which the sexes differ, it can be argued that differences in mathematics achievement may be due to differences in spatial ability. Recent studies in brain hemispheric specialization show differences in the sexes with females less well lateralized than males (McGlone and Davidson, 1973; Wada, 1972; Knox and Kimura, 1975; Harris, 1973). There may indeed be differences in brain organization. While the relationship the authors have demonstrated is surely a factor in sex

differences in mathematics achievement, other hypotheses must be considered.

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SE 015 929

ELEMENTARY AND MIDDLE SCHOOL CHILDREN'S COMPREHENSION OF EUCLIDEAN TRANSFORMATIONS. Kidder, F. Richard. Journal for Research in Mathematics Education, v7 n1, pp40-52, January 1976.

Based on a doctoral dissertation directed by Leslie P. Steffe, University of Georgia.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Sally H. Thomas, Orange Coast College.

1. Purpose

To investigate 9-, 11- and 13-year-olds' ability to perform Euclidean transformations (at Piaget's representation level) while holding length invariant.

3. Rationale

Transformational geometry has been recommended for inclusion in elementary- and middle-school mathematics programs, but little research exists concerning children's spatial ability to perform Euclidean transformations. Piaget has found that the earliest spatial concepts are topological; projective and Euclidean concepts develop later and equilibrium is achieved when the child is about 9 or 10. Thus middle-school children should have the ability to perform Euclidean transformations as well as to conserve length. Piaget has hypothesized that adolescents' cognitive operations have a mathematical group structure. Since transformations also form a group, studying middle-school children's abilities to perform transformations allows an additional opportunity to study the development of children's thought processes. This study investigates students' abilities to perform a group of transformations on representations of a triangle while holding length invariant. It also allows the experimenter to observe the level of students' cognitive operations. The tasks used for this investigation were modeled after those described by Piaget and Inhelder in The Child's Conception of Space.

3. Research Design and Procedure

There were 90 subjects, 15 each aged 9, 11, and 13, from each of two Georgia schools. Teachers were asked to exclude students who were in the lower one-fourth general-ability level. A pencil-and-paper spatial analogies pretest was administered. The Otis-Lennon Mental Ability Test Form I was given post hoc as a measure of general ability; means for each age group were 43.6, 52.9, and 46.1, respectively.

Each subject was individually given an operational definition of "slides", "flips", and "turns": a rigid model of a triangle, a motion indicator (slide-arrow, flip-line, or turn-arrow), and a congruent copy of the original triangle were used to illustrate the motion and the triangle's resulting position. After this instruction, 72 of the 90 participants

were found to be operational or able to perform three consecutive transformations of each type.

Immediately after the instruction, the subjects were given the transformational test that examined performance at Piaget's representational level. This test consisted of four tasks of each of the individual motions (slide, flip, or turn), six tasks requiring composition of transformations (e.g., slide-turn or slide-slide), and three tasks requiring the inverse of a transformation (e.g., inverse of a turn). Each task also involved constructing a triangle congruent to the original triangle from a set of sticks congruent to the original sides and distractors of different lengths.

4. Findings

The analysis of variance (motion x age) showed that motion was a significant factor in each test or type of task (pretest, operational definition, individual motions, composition, inverse motion). Age was a significant factor only on the pretest and individual motions task. Slides were easiest for the students to perform; performances on flips and turns did not differ significantly. The lack of age significance on most tests was apparently due to the poor performance by subjects of all ages on the transformation tasks. For example, of the 180 compositions required of each age group, only 10 (5.6%) were performed satisfactorily by the 9-year-olds, 12 (6.7%) by the 11-year-olds, and 19 (10.6%) by the 13-year-olds. The subjects performed best on the slide task; however, 56% to 80% of the subjects in each age group were unable to perform even this "easiest" task.

The pretest scores for perceptual ability did not correlate highly with performance on the transformations. However, there was a definite relationship between Otis-Lennon Mental Ability Test scores and the number of transformation tasks performed satisfactorily.

The inability of the subjects in each age group to perform the individual motions precluded examining the relationship between cognitive structure and the structure of the mathematical content.

5. Interpretations

The results of this study appear to contradict Piaget's contention that children have the framework for understanding Euclidean systems by about age nine. An error analysis revealed that about 70% of the subjects in each age group failed to conserve length under individual motions; that is, they did not construct images congruent to the given triangle. Piaget and others have found that length is conserved by 6- to 8-year-olds when they are shown two sticks of equal length and then asked which is longer after one stick has been moved a centimeter. However, the conservation of congruent triangles, required in this study, may not occur until a later age.

The similarity of the results between age groups on most tasks may indicate that few of the children, even in the oldest group, had achieved formal-operational thought. One general property of this stage is the

ability to separate out factors not observed directly and to hold one factor constant while determining the causal factor of another. In the transformation tasks, the subject had to preserve the size and the shape of the triangle. He also had to recognize that the angles made by two sides of the triangle with an imagined line (through their common vertex and parallel to the third side) remain constant while the imagined line and the triangle move through a slide, turn, or flip. A child who has not reached the stage of formal operations tends to shift his attention from one variable to another in performing transormational tasks, as did the subjects in this study. Thus, the results of this study may indicate that these subjects had not attained the expected stage of cognitive thought, rather than refuting Piaget's theory of spatial development.

Critical Commentary

This study provides good tests of children's abilities to understand transformations and an interesting application of Piaget's theory. Unfortunately, the results of the general ability test - with 13-year-olds scoring lower than 11-year-olds - create questions about the sample used in this study and thus about the results. In planning the study, it is too bad that the investigator relied on subjects' grade placement as an indicator of cognitive development and did not independently assess the subjects' operational stages. It would be instructive to see a replication of this study using a more appropriate sample.

The author does not describe the methods he had planned to use to compare transformational group structure and the cognitive structure of the middle-school child. Even though this comparison was not possible, it would have been instructive to know what he had planned.

Should all thirty students have been included in each age group in the ANOVA since 18 subjects were not "operational" after the initial instruction? The omission of percentages in section 2 of Table 3 is annoying, but they can be reconstructed.

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EJ 091 685 SE 010 287
ADOPTING A TEXTBOOK, OR: RESEARCH CAN HELP! Klein, Alice E. School Science and Mathematics, v73, pp743-746, December 1973.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert B. Ashlock, University of Maryland.

1. Purpose

To provide a data base for selection of elementary school mathematics texts within a school district.

2. Rationale

Textbook selection often rests on subjective or at least unexplicated bases. However, many educational researchers would welcome the opportunity to work with school districts designing and conducting a study. Thereby, decisions can be based on data and the rationale for decisions can be articulated better.

3. Research Design and Procedures

Ten classes per grade (one through six) were randomly selected from all classes in the school district. In all, 60 teachers and more than 1700 students participated in the study.

Each pupil was classified as either above average or below average reading level according to a grade equivalent score on standardized tests. A teachers' committee selected five mathematics text series. Instruction in each of the experimental classes was based on one of the five text series. Scores on modern and traditional mathematics achievement tests collected at the end of the school year were used as the dependent variables. Teacher evaluations of the text used were also collected.

Two-way analyses of variance were applied to each criterion measure using reading level and texts as the independent variables for each grade.

4. Findings

No text series proved to be clearly better than any of the others for all grades when both modern and traditional mathematics tests were used as the criteria. Only six of the 12 analyses of variance produced statistically significant effects. An examination of the results showed two of the five texts to be generally more effective than the others. Teachers' evaluations overwhelmingly indicated a preference for one of these two.

5. Interpretations

The study illustrates how a school district might gather data to use in making textbook decisions.

Critical Commentary

This report illustrates the potential of a process for making more informed decisions about the use of different mathematics textbooks. As an illustration of a process, it is a welcome contribution to the literature of mathematics education. The researcher's involvement of school district personnel is especially commendable.

However, the report itself is so incomplete as to make adequate judgments about the study almost impossible. The reader is given only a brief sketch of the research.

For example, although we know that five different text series were used, and ten classes were used at each grade level, we are left to wonder if possibly two classes per grade level were used for each text series. The duration of the treatment is not even mentioned.

References to the tests employed are also inadequate. The standardized tests used for reading level are not identified, nor are the modern and traditional mathematics achievement tests. In fact, it is not clear whether these tests were used in conjunction to determine one score, or if separate computations were made.

The researcher does not list the 12 hypotheses tested, nor does she identify the level of confidence for the six hypotheses reported to be significant.

The study was presented at the annual meeting of the National Council of Measurement in Education in 1972; thus, a more complete report may be available. Unfortunately, no reference is made to such a report.

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SE 016 245

AN ANALYSIS OF CHILDREN'S WRITTEN SOLUTIONS TO WORD PROBLEMS. Knifong, Dan J.; Holtan, Boyd. Journal for Research in Mathematics Education, v7 n2, pp106-112, March 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by A. Edward Uprichard, University of South Florida.

1. Purpose

To determine and classify the errors made by children in attempting to solve mathematics word problems, so that teachers can have a more factual basis for planning instruction.

2. Rationale

Previous research-based explanations have not satisfactorily explained the nature of children's word problem difficulties. Nevertheless, the teaching of reading or reading-related skills is persistently advocated for improving effectiveness in solving mathematics word problems. Therefore, there is a need to determine the extent to which reading difficulties contribute to errors in solving word problems.

3. Research Design and Procedure

Thirty-five children selected from three sixth-grade classes in a predominantly white, middle-class, urban elementary school attempted the word-problem section of the Metropolitan Achievement Test (1958) and their written work was analyzed to determine the typical errors in 470 incorrectly solved problems (45 percent of 1,050 attempted, 30 problems x 35 Ss). The problems on the test were retyped to allow work space beside each item. The 35-minute time period was extended to 55 minutes. The only directions were to complete as many problems as possible and to show the work used beside each problem.

Errors were classified into two major categories: (I) clerical and computational errors and (II) "other errors" such as use of the wrong operation, lack of any response, and the special case of average- and area-item errors.

"Clerical errors" consisted of copying errors or writing the correct answer in the wrong space. "Computational errors" with whole numbers and/or fractions included problems that were calculated not at all, partially, or incorrectly. In each case it was clear that the child missed the problem due to faulty computations. Errors involving the use of common units of measurement (minutes, hours, pounds, pints, dozens, dollars, etc.) were included as a subclassification of computational errors.

Under category II errors, if the student's work on all items showed some attempt at computing the average or area (not using the proper procedure) or if the student made a sufficiently accurate guess to indicate

central tendency, the error was classified under "average and area errors". Problems worked using an incorrect operation or an incorrect or incomplete series of operations were classified under "use of wrong operations". A "no response" error occurred if the student did not respond to an item. If there were no clues as to why an incorrect response occurred, the error was classified under "incorrect responses offering no clues".

Two independent raters assigned each incorrectly solved problem to one of the assigned categories and the mean of the two frequencies for each type was reported. There were only 57 disagreements as to how to categorize a particular error; the reliability of the mean score was .92. To prevent multiple listings, each incorrectly solved problem was assigned to only one type of error. If more than one error was made, the problem was assigned to the error type farthest down the list. This led to clerical and computational errors being underrepresented, while the category "other errors" was overrepresented.

4. Findings

Clerical and computational errors accounted for 52 percent of the incorrectly solved problems with computational errors alone accounting for 49 percent. Reading and other factors that might have caused errors accounted for the remaining 48 percent. See Table 1 for a more complete breakdown of results.

Table 1

Frequency and Percentage of Total Errors by Type

	Number	Percentage of Total Errors	Totals	
I. Clerical and computational errors				
A. Clerical errors	12.5	3%		
B. Computational errors				
1. with whole numbers	61.5	13%		
2. with fractions	88.5	19%		
3. of standard notation and units	80.5	17%		
Total clerical and computational errors			243	52%
II. Other errors				
A. Average and area errors	22.5			
B. Use of wrong operation	30.0			
C. No response				
1. but went on to other problems	58.5			
2. did not attempt any further problems	85.5			
D. Erred responses offering no clues	30.5			
Total other errors			227	48%
Total incorrectly solved problems from the 1,050 attempted (35 students x 30 problems)			470	100%

5. Interpretations

The data do not relegate reading to a minor role as a source of difficulty. However, the authors, based on the results of their study and the study by Barlow (1964), concluded that it is difficult to attribute major importance to reading as a source of error in word problems. Three hypotheses were presented in attempting to determine the extent to which reading difficulties figured as a source of error in 48 percent of the incorrect problems: (a) Reading difficulties claimed all of the 48 percent categorized as nonclerical and noncomputational errors. (b) Inefficient reading skills slowed students so they were unable to complete the test within the extended time. Thus reading might account for about 18 percent of the incorrectly solved problems. Or, (c) poor reading skills accounted for none of the errors, since the mean of 85.5 instances of problems left blank at the end of the test could be explained by noting that this achievement test, like others, had its most mathematically difficult problems toward the end.

Since computational errors accounted for so large a percentage of incorrect problems, lessons on improving computational skills were strongly recommended as a teaching strategy.

Critical Commentary

This study can be characterized as clinical in nature. Although there was no experimental research design or traditional statistical treatment of data, the investigators attempted to organize procedures and analyze the data in a systematic fashion. Clinical studies yield data from which one can generate hypotheses rather than make general inferences. Investigators employing clinical methods must keep this in mind when interpreting findings.

An analysis of the results of the present study can be helpful in generating some important hypotheses with respect to difficulties which learners experience in solving mathematics word problems. However, before formulating these hypotheses, some issues related to this study need to be examined.

1. Classification of Computational Errors. The authors state that 49 percent of the errors made by the students in solving word problems were computational errors. I find it difficult to justify errors of standard notation and units (17 percent of total errors) as a subset of computational errors.

2. Readability of Word Problems. The authors state that it is difficult to attribute major importance to reading as a source of failure. What was the average reading level of the sixth-graders used in this study? What was the readability level of the word problems attempted? This writer applied the Fry Readability Formula (1968) to the first four word problems (combined) of the Metropolitan Achievement Test (1959). If

one considers multidigit numerals as single syllable words, the readability level is fourth grade. Further, the MAT Test Manual states that in the word problem section "the reading load has been kept at the lowest possible level (p. 4)".

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EJ 090 168 SE 010 239

WHAT CAN A TEACHER LEARN ABOUT A PUPIL'S THINKING THROUGH ORAL INTERVIEWS?
Lankford, Francis G., Jr. Arithmetic Teacher, v21, pp26-32, January 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Cecil R. Trueblood, The Pennsylvania State University.

1. Purpose

To record and analyze the thinking and the computation strategies of seventh-grade pupils.

2. Rationale

Traditionally teachers have examined pupils' written computation work on paper-and-pencil tests in order to diagnose computation errors and prescribe correctives and practice materials. The written records pupils make are by themselves insufficient for detecting the reasons for pupils' errors. Recent and past research studies indicate that knowledge of pupils' thinking as they compute can be determined by carefully conducting individual interviews and that this information in addition to results from written records can be used to improve pupils' computation skill.

3. Research Design and Procedure

During the 1972 school year 176 seventh-grade pupils in six different school systems in several states were individually interviewed with regard to their written computation performance. In each school system, all pupils in one intact class in a single school were used. The interviews were conducted in a room where a single pupil and the interviewer could work undisturbed. A standard set of introductory procedures and directions were used.

Each pupil was given a set of standard computation exercises and asked to do them as he or she usually did, but to "say out loud your thinking as you compute." The exercises involved the four operations with whole numbers and fractions. Each interview lasted about 40 to 50 minutes. The pupils were not hurried; they simply completed as many exercises as they could in the time alloted. A verbatim record of each interview was made on cassette tape and later analyzed to determine the computational strategies used. A computational strategy was defined as the pupil's pattern of thinking as he or she computed.

The interviewer did not engage in any teaching. That is, he was careful to avoid giving cues, asking leading questions, or commenting about the correctness or incorrectness of answers or procedures. The pupils were not interrupted as they computed. At points where pupils hesitated or did mental calculations, they were asked to give an oral report of their thinking AFTER finishing the exercise. When pupils discovered errors on their own, they were directed to mark through the error and write the correction to the side of the example. Pupils were not asked to repeat an example if they made an error unless the interviewer wanted to understand more clearly what the pupil did.

4. **Findings**

Some specific computational strategies were identified as a direct result of using the interview technique. The most frequently used are given below.

Operations With Whole Numbers

1) Counting was the most frequently used strategy and many pupils used some means of "keeping count" of the counting. Forty-five used their fingers, fourteen made motions in the air, and fourteen used dots or scratch marks.

2) A high number of pupils (74) used doubles to add. In multiplication pupils derived unknown combinations from known ones by counting.

3) Only 27 pupils used the inverse relationship between addition and subtraction to guide their thinking. A large number (123) regrouped quite mechanically when subtracting. The wrong order was also often used in subtraction.

4) In division exercises there was little evidence in the pupils' vocabulary that they thought of division as the inverse of multiplication. Much more frequently (155 cases) students used such phrases as "27 divided by 81" or "27 goes into 81."

Operations With Fractions

1) All addition with fractions examples were arranged horizontally. The practice of adding numerators to numerators and denominators to denominators to get the sum was used by 62 pupils. A smaller number (10) added numerators and placed the sum over the larger denominator of the fraction addends.

2) Three of the subtraction with fractions examples were arranged horizontally and two vertically. The exercise 9 2/3 - 5 7/8, which required renaming (e.g., 9 16/24 - 5 21/24), caused considerable difficulty for some pupils (12). Thirty-eight correctly "borrowed 1 from 9" and rewrote it as 24/24 and added this to 16/24, but 12 students "added the 1 to 16" making it 26, thereby getting 26/24.

3) In multiplication with fractions 31 pupils used the following incorrect computation strategy: for 2/3 x 3/5 they wrote equivalent fractions with a common denominator and then multiplied the numerators of the equivalent fractions for a product (e.g., 2/3 x 3/5 = 10/15 x 9/15 = 90/15).

4) In division with fractions 36 pupils wrote equivalent fractions with a common denominator and then divided the numerator (e.g., 7/8 ÷ 2/3 = 21/24 ÷ 16/24 = 1 5/16). Many who used this strategy made the final division incorrectly and wrote the quotient as 1 5/24.

In general there were some marked observable differences between the computational practices of good and poor computers. These are:

Good computers -

a) knew the basic combinations and did not need to derive them by primitive methods such as counting.

b) tended to follow conventional algorithms rather consistently. They remembered what they had been taught to do and followed the orthodoxy of classroom and textbook quite closely.

c) used pencil and paper more than would appear necessary, especially with simple exercises.

d) appeared to be much less dependent on the vertical or horizontal arrangement of an exercise to provide a clue to the appropriate algorithm than did the poorer computers. (This was especially noticeable with fractions.)

e) did much better than the poor computers in the final group of eight comparison exercises, both with their choices and their reasons.

f) seemed to have better memories. For example, once they had identified an exercise as requiring a certain algorithm, they were quite likely to remember and use it correctly.

g) appeared to sense more often than poor computers when an answer was wrong, and proceeded to make corrections.

h) often ran ahead of their words or pencils in their thinking.

i) tried out computations mentally and quickly, as in finding a common denominator or a quotient digit.

Poor computers -

a) relied heavily on a few retained facts such as doubles or products with 5 as one factor, from which they derived unknown combinations. They made extensive use of counting to make combinations. Their stock of whole-number facts was limited.

b) often made errors in whole number operations when their counting or other derivations of unknown combinations became too involved for their short memory spans.

c) had much more trouble with fractions than with whole numbers. Primitive methods such as counting are not useful with fractions as with whole numbers, although a few pupils tried to think with "pieces of pie" in operating with fractions.

d) had difficulty remembering the conventional operational algorithms, especially in fractions. Moreover, they had difficulty in matching those they did remember with the right exercise.

So they devised simple, and what seemed to them as obvious, procedures such as adding numerators and then adding denominators for the sum of two simple fractions:

e) often switched to something else that would produce an answer, when they encountered difficulty with an improvised algorithm, regardless of how remote from the proper procedure it might be.

f) tended to rely more on mechanical aids to memory.

g) often supported by a reason, even if faulty, what would appear to be careless errors.

h) had great difficulty with long division as in $74\overline{)6484}$. They would make several trial multiplications on the side in an effort to find a quotient digit. The factors they chose to multiply the divisor by were often quite arbitrarily chosen.

i) often did not hesitate to reverse minuend and subtrahend in subtraction.

j) frequently confused 0 and 1, as in 15 3/4 + 3/4 = 15 0/4.

k) were quite likely to be confused in arranging the partial products of multiplication, especially when the factors contained zeroes.

Finally, the study revealed in a dramatic way that pupils varied widely in the computational strategies they employed for operations with both whole numbers and fractions.

5. Interpretations

These findings demonstrate that the interview technique does reveal aspects of a pupil's computational strategies that cannot be obtained by traditional paper-and-pencil tests alone. There is a distinct difference between the computation strategies of good and poor computers.

Critical Commentary

This study demonstrates that the interview technique can provide teachers and researchers with useful information about students' thinking relative to their computational skill and strategies. The basic investigative procedures used in this study are sound and the author's interpretation of the results are accurate. The comparisons made between good and poor computers are not supported by the data reported in this article but they are supported by the data in the full report of the research project conducted (and cited) by the author.* Two questions that can not be answered from this study are:

(1) Would other teachers or interviewers interpret the interview data the same way as the investigator? The investigator was a mathematics educator with many years of experience. (2) How much training would beginning teachers require so they can get unbiased data?

Other questions that should be pursued in further research are:

1) What teacher training material will help prospective teachers learn how to interview and interpret the results?

2) Will students given individual help based on interview data dramatically improve their computation skill?

3) What type of classroom management system must be used to make the interview procedure practical in terms of time, personnel, and materials?

4) How should interviewers be trained so they could obtain similar findings?

5) How should teachers and curriculum writers use the type of data collected in this study to improve instruction in computation?

6) Why do seventh graders use so many inefficient computation strategies?

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*Lankford, Francis G., Jr. Some Computational Strategies of Seventh Grade Pupils. ERIC: ED 069 496. (1972)

SE 016 241

MODELS AND APPLICATIONS AS ADVANCED ORGANIZERS. Lesh, Richard A., Jr.; Johnson, Howard. Journal for Research in Mathematics Education, v7, pp75-81, March 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by John C. Peterson, Eastern Illinois University.

1. Purpose

This study had three purposes:

1. To investigate the relative effectiveness of four different types of advanced organizers when they were used to introduce a thirty-minute self-instructional unit about elementary concepts of motion geometry.

2. To compare children who received an advanced organizer with children who received a post organizer.

3. To compare children who worked on the organizer in small groups with children who worked individually.

2. Rationale

No rationale was given in the article; the previous article in the journal (Lesh, 1976) contains a partial rationale for this study, however. It was a replication and extension of Johnson's doctoral dissertation. Changes were made by (a) increasing the number of students, (b) including seventh- as well as fourth-grade students, and (c) not using a control group.

3. Research Design and Procedure

A 2x2x2x2 factorial design was used. The independent variables were type of organizer (advanced vs. post), type of example (model vs. applications), number of examples (one vs. several), and number of children working together on the organizer (one vs. several). At each grade level, 15 children were randomly assigned to each of the 16 treatment groups. All treatment groups received the same self-instructional unit. Ss were 240 fourth graders and 240 seventh graders selected from four North Chicago schools.

There were four basic types of organizers. One type used several simple applications to introduce intuitively the ideas that were in the unit; another used several concrete models; a third used a single application; and the fourth used a single concrete model. A situation was judged to be an application if a system of relationships had to be imposed on a set of material in order to illustrate a given idea. However, if the relevant system of relationships was already inherent in the materials, then it was judged to be a model. Directions for the organizers were given through the use of a video-taped presentation. For the organizer sections in which small groups of four children worked together, the

children were encouraged to discuss the examples they were given. Advanced organizers differed from post organizers only in that the former were given before the unit and the latter were given after the unit.

The basic unit was written as a 30-minute self-instructional unit and presented on a sequence of 5 x 8 index cards. Materials were taken from the Johnson study or adapted from the UICSM book on slides, flips and turns. Simple geometric figures (usually circles and polygons) were used to illustrate slides, flips and turns. A red dot was placed on the figure to indicate a rotation point and a red line was used to indicate a "flip" line.

The 15-minute posttest was constructed using many of the items that had been used on Johnson's study. A Kuder-Richardson Formula 21 reliability of .85 for the posttest was established. The posttest used in the Johnson study consisted of two subtests. The "egocentrism" subtest consisted of items that would tend to be missed by a child who read incorrect or irrelevant information into the examples and definitions that were given. The "centering" subtest consisted of items that would tend to be missed by a child who failed to read important information in the examples and definitions that were given because he focused his attention only on the most obvious aspects of these examples and definitions, ignoring other important but less obvious features. In order to follow the Johnson study, an attempt was made to classify posttest items according to whether they were egocentric items or centering items. No reliability coefficients for these two subtests are given. The posttest was given immediately after the treatment was completed. It took about one hour to finish both the treatment and the posttest.

4. Findings

Significant differences were reported at the fourth-grade level for application vs. models ($p < .05$), one example vs. several ($p < .05$) and advanced organizers vs. post organizers ($p < .01$). At the seventh-grade level significant differences existed between the advanced organizer and post organizer groups ($p < .01$) and small groups vs. individuals ($p < .05$). No significant differences were found between the egocentrism items and the centering items. In fact, a correlation of .61 was obtained between scores on the egocentrism subtest and the centering subtest.

5. Interpretations

For both the fourth graders and the seventh graders, the advanced organizer groups scored significantly higher than the post organizer groups. "However, it is important to point out that the posttest was given immediately after the instructional unit was completed."

The high correlation between scores on the two subtests seems to indicate that a child's tendency to read irrelevant information into illustrations may be closely associated with the tendency to neglect important information that should be read out of illustrations that are given.

The effect of receiving organizers with several types of examples rather than one example seemed to become less important for older children.

The effect of group instruction was significantly better than was individual instruction for seventh graders. That the two groups were equally effective with fourth graders conflicts with the results of the Johnson study where the individual treatment was more effective.

The fact that the models organizers were more effective than the applications organizers for fourth grade and not for seventh may be a result of the inability of fourth graders to impose the relevant relationships in the models on the materials.

Critical Commentary

The authors should be congratulated for replicating and extending their previous study. It would have been helpful if they had provided a rationale for their study. Why did they feel the study should be replicated? Why did they include seventh graders? Why was there no control group?

Directions for the organizers were given via a video-taped presentation. The statement quoted in the interpretation section indicates that the directions for the post organizer group were given prior to the instructional unit. Could the half-hour during which the instructional unit was given have caused Ss who were given post organizers to forget the directions enough to interfere with the effect of the organizers?

Approximately 25 percent of the paper was spent discussing the egocentrism and centering subtests. However, this was not one of the objectives for the study.

While a reliability coefficient is given for the posttest, no means or standard deviations are given for the groups. This information should have been included in the paper. The number of items in the posttest and each subtest should also have been reported. What were the reliabilities of the subtests?

Some minor questions could be asked. How many applications or models were used in the groups that received several? Ss worked together in groups of four. If the groups were disjoint how was this done with an n of 15?

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Lesh, R. A., Jr. An Interpretation of Advanced Organizers. Journal for Research in Mathematics Education, v7, pp69-74, March 1976.

ED 115 490 SE 019 853

STUDY IV - A COMPARATIVE STUDY OF THREE REMEDIAL METHODS OF INSTRUCTION FOR TWO-DIGIT ADDITION WITH REGROUPING AT THE SECOND GRADE. Maalouf, Munif; Heck, William; and Romberg, Thomas A. In Romberg, T. A. (Ed.), Four Studies on Aspects of Assessing Computational Performance. Technical Report No. 297. Wisconsin Research and Development Center for Cognitive Learning, The University of Wisconsin - Madison. July 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward M. Carroll, New York University.

1. Purpose

The study considers the effect of three methods of remedial instruction to answer these questions:

a. Should children receive remedial instruction within the same context under which they were initially instructed?

b. Are there motivating devices available which might make the task of remediation of skills easier?

c. Should meaning and understanding be sacrificed at some point in favor of a rote approach to the instruction of computational skills?

2. Rationale

The authors list several factors which contribute to differences in understanding of a given mathematical unit or skill, including methods of instruction that do not take into account:

a. the student's capabilities or learning needs

b. the student's need to manipulate physical materials in order to master a given skill (Schulz, 1972)

c. the student's need to learn a rote method before mathematical understanding (Anderson, 1949)

The general assumption is that there are many strongly held opinions on the subject, but there "seems to be little clear evidence regarding which children should learn which mathematics in which ways."

3. Research Design and Procedure

A pretest of 25 items was administered to all 89 second-grade students who could profit from remedial instruction on two-digit addition with regrouping. Basic instruction on the topic had been given to all students through expanded notation progressing to the traditional carrying approach. The test consisted of four parts:

a. Ten "teen facts"

b. Five two-digit addition problems with no regrouping

c. Five two-digit addition problems with regrouping

d. Five three-digit addition problems with regrouping

Five forms of the pretest, consisting of problems randomly generated by a computer, were distributed to the classroom teachers, who administered and supervised the test.

An "ideal" score pattern (for the purposes of the study) was defined in terms of correct scores as follows:

Part 1	Part 2	Part 3	Part 4
10 correct	5 correct	0 correct	0 correct

Sixty-three students solved all 10 problems correctly in Part 1; 74 solved all 5 problems correctly in Part 2; 30 students did not solve any problems in Part 3; and 33 students solved no problems in Part 4. Fifteen students whose scores approached the "ideal" score pattern were selected and randomly assigned to three groups in such a way that each class was represented in each group.

Instruction for each of the three groups was carried out by an investigator during two periods, one of 30 minutes and one of 15 minutes. The purpose of the instruction was to teach solving two-digit additions with regrouping, using one of the following methods:

First group:	The calculator method
Second group:	The subsums method
Third group:	The traditional carrying method

A 20-minute posttest was administered to the 15 students. The test consisted of two parts: (1) ten two-digit addition problems with regrouping and (2) five three-digit addition problems with regrouping. As an afterthought a retention test, identical in form and number of problems to the posttest, was administered after 11 days.

The means and standard deviation of correct responses were computed and data were subjected to a one-way analysis of variance.

4. Findings

a. On the posttest the three methods of instruction did not differ significantly.

b. A marked trend favored the third method (traditional carrying) for remedial teaching of solving two-digit addition with regrouping.

c. Transfer to three-digit addition with regrouping was better with the third method, while methods 1 and 2 resulted in very little transfer.

d. On the retention test, scores of the three groups differed significantly in favor of the third method.

5. Interpretations

The findings reported in this study tend to suggest that the three methods of instruction used can provide an increase in achievement of two-digit addition with regrouping. The instruction did not stress any relationship between meaning and comparison. The traditional carrying method yielded better achievement on both the post and retention tests and contributed more transfer of learning to the three-digit addition with regrouping than did the other two instructional methods. The calculator method resulted in the least achievement and transfer. The significant difference in the retention test was probably due to the low scores of the students in the calculator method. The investigators suggested that the trend in favor of the third method could be attributed to the background of the students, to the method itself, or to the fact that the children's initial instruction was with this same method. Further, they suggest a replication of this study and a study to investigate the effect of rote instruction on subsequent understanding of the unresolved algorithms.

Critical Commentary

The authors reported that "there seems to be little clear evidence regarding which children should learn which mathematics in which ways." The entire question of the desirability of emphasizing the development of computational skills at the expense of mathematical understanding is a value judgment that teachers must make. The easy way out of this problem is the return to the "Rule-Example-Drill" syndrome of the past. In this study the "R-E-D" technique is used in two of the remedial instructional methods for solving two-digit addition with regrouping. As a result there seems to be a bias in favor of the traditional method. In fact, the study was designed to answer only question number 3: Should meaning and understanding be sacrificed in favor of a rote approach to the instruction of computational skills? The real findings of the study are that more drill in a particular approach tends to increase computational skills. This finding can be deduced from the given data that "all students had received basic instruction in two-digit addition with regrouping through expanded notation progressing to the traditional carrying approach."

The manner in which the hand-held calculator was used to develop problem number 2 is questioned. It is generally agreed that children must play with and investigate instructional aids to eliminate the novelty before structured lessons begin; this is especially true for seven-year-old children. The number of practice problems selected is an indication of this difficulty. Group 1 solved only 160 problems as compared to 437 problems solved by group 3 during the instruction periods.

The answers to the following questions would have been other useful additions to the study and report:

1. Was 15 students a valid sample of the population?

2. Were two periods, one of 30 minutes and one of 15 minutes, sufficient instructional time? Were these the same day, the same month?

3. How soon after instruction was the posttest administered?

4. What was the time duration of the retention test? How was 20 minutes judged to be adequate for these children?

The questions suggested in this study need to be answered. I would like to see investigations conducted to answer them.

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SE 015 928

A TEST WITH SELECTED TOPOLOGICAL PROPERTIES OF PIAGET'S HYPOTHESIS CONCERNING THE SPATIAL REPRESENTATION OF THE YOUNG CHILD. Martin, J. Larry. Journal for Research in Mathematics Education, v7 n1, pp26-38, January 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by M. C. Mitchelmore, Ministry of Education, Kingston, Jamaica.

1. Purpose

To test Piaget's hypothesis that the four-year-old child's representational space is predominantly topological in nature.

2. Rationale

On the basis of experiments on the identification of objects by touch and the copying of figures by drawing, Piaget & Inhelder (1967) proposed that topological concepts develop prior to Euclidean concepts in the child's representational space. They state that topological features are most salient between ages 3 and 6, and that it is not until age 7 or later that distinctions are made on the basis of Euclidean characteristics. Although their results have been confirmed by several replicators, alternative tasks are needed to test the generality of the hypothesis. In the present study, subjects were presented with a geometric figure either tactually or visually, and forced to choose between a copy which preserved all topological but not all Euclidean properties and copies which preserved most Euclidean properties but not all topological properties. If four-year-old children's representational space is predominantly topological, they would be expected to prefer the homeomorphic (topologically equivalent) copy. Older children would be expected to reverse this preference.

3. Research design and procedures

Six model designs were drawn on file cards and also made of wire glued to masonite. Three transforms of each model were prepared: Copy A was homeomorphic to the model, but lines and curves were made slightly uneven and corners were rounded off. The other two transforms were non-homeomorphic pseudo-Euclidean copies; linearity and angularity were preserved, but small gaps were introduced to change the connectedness (copy B) or openness (copy C) of the figure.

A heterogeneous sample of 90 subjects of both sexes (30 each at ages 4, 6 and 8) were tested. Subjects were first presented with a model figure to examine; it was then taken away and the child asked to select the copy "most like" the model and the "worst" copy of the model. All six figures were used in each of four testing modes, given in two sessions as follows:

Session I

Mode (1) Tactile examination, tactile selection

Mode (2) Visual examination, tactile selection

Session II

Mode (3) Tactile examination, visual selection

Mode (4) Visual examination, visual selection

In addition, subjects made drawings of the last model examined tactually in mode (1) and the last model examined visually in mode (4).

Two scores were obtained for each test mode: the number of times copy B or copy C was chosen as "most like" the model, and the number of times copy A was chosen as the "worst" copy. Both scores were tested for deviation from chance means using one-tailed t-tests. The scores were also subjected to univariate two-way analyses of variance (age x mode) and post hoc analyses using Duncan's Multiple Range Test.

Drawings were categorized as homeomorphic or non-homeomorphic to the model and were also assessed for the preservation of linearity and curvature. The proportions of drawings which fell into each category under each of the two examination conditions were inspected for age trends.

4. Findings

(a) "Most like" scores were significantly above chance in modes (3) and (4) at all ages ($p < .01$) and in mode (2) at age 4 ($p < .05$). No other "most like" scores were significantly above chance.

(b) "Worst" scores were significantly above chance in modes (3) and (4) at all ages ($p < .01$) and in modes (1) and (2) at age 6 ($p < .05$) and age 8 ($p < .01$). Scores in modes (1) and (2) at age 4 were not significantly above chance.

(c) "Most like" scores showed a significant mode effect ($p < .01$) but no significant age effect or interaction. Scores in modes (3) and (4) were significantly higher than in modes (1) and (2).

(d) "Worst" scores showed significant mode ($p < .01$) and age ($p < .05$) effects but no significant interaction. Scores in modes (3) and (4) were significantly higher than in modes (1) and (2), and scores of four-year-old children were significantly lower than scores of six- and eight-year-old children.

(e) For drawings made after tactile examination, the proportion of homeomorphic copies rose from 10% at age 4 to 70% at age 8. For drawings made after visual examination, the proportion of homeomorphic copies rose from 40% at age 4 to 90% at age 8.

5. Interpretations

Piaget's hypothesis seems to predict that both "most like" and "worst" scores of four-year-old children would be significantly below chance in all four modes. In fact, five of the eight means were significantly above chance and the remaining three were not significaltly different from chance. The prediction was therefore not confirmed.

The hypothesis would also predict a significant increase in both scores between age 4 and age 8. However, the age effect was significant only for the "worst" scores.

A third prediction is that drawings made by four-year-old children would correctly depict the topological properties of a figure more frequently than the Euclidean properties. This prediction also received no support from the drawings analyzed in this study. Closer inspection suggested that the increase in the proportion of homeomorphic drawings from age 4 to age 8 was a result of the increasing coordination of Euclidean and projective concepts between those ages.

The study does not support the hypothesis that the representational space of four-year-old children is predominantly topological, nor the hypothesis that topological concepts develop prior to Euclidean and projective concepts.

Critical Commentary

Since the hypothesis under investigation was that topological concepts predominate at age 4, it would have been easier to interpret the results of this study had the author used topological preference scores instead of his "most like" and "worst" scores, which indicate Euclidean preference. This procedure might also have led him to avoid the error of using one-tailed tests to test the significance of deviations in the opposite direction to that predicted by hypothesis. A few other small technical criticisms (including a misprint in Table 4) could be made, but they do not bear substantially on the overall results.

The author's discussion of his results is disappointingly short by comparison to the introduction. Relevant studies of Fisher (1965), Esty (1970), and Cousins and Abravanel (1971) are not explored: Why was the non-homeomorphic copy preferred more often under visual than under tactile selection? Why were homeomorphic copies drawn more often after visual than after tactile examination? Were these order effects? Why were the results different for "most like" and "worst" scores? This paper would have been even more stimulating had Martin felt freer to speculate on the implications of his results.

However, the main problem with this study is to decide whether the results actually do contradict Piaget's hypothesis. Piaget does not claim that four-year-old children only have topological concepts in their representational space, only that "in the case of conflict [topological relationships] prove stronger." His hypothesis therefore does not predict that four-year-old children will select any homeomorphic copy in preference

to any pseudo-Euclidean copy. One can easily imagine a homeomorphic copy which is so convoluted that no child would select it--or a copy which is so little distorted that many adults would select it. The pseudo-Euclidean copy can also vary in its degree of distortion according to the number, position and size of the gaps introduced. How did the experimenter decide the appropriate degree of distortion for each type of copy? In order that a preference for the non-homeomorphic copy shall unequivocally indicate the salience of Euclidean characteristics, it is necessary to demonstrate that the distortion in the homeomorphic copy is in some sense less than the distortion in the non-homeomorphic copy; no such argument is advanced in this paper. Martin's results may merely indicate that the distortion in his homeomorphic copies was greater than his non-homeomorphic copies. Further analysis suggests that the method employed in this study is inherently unsuited for testing Piaget's hypothesis.

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USING ELECTRONIC PROGRAMMABLE CALCULATORS (MINI-COMPUTERS) IN CALCULUS INSTRUCTION. O'Loughlin, Thomas. American Mathematical Monthly, pp281-283, April 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Larry K. Sowder, University of Oregon.

1. Purpose

The purpose of the study was "to investigate the effect of the incorporation of an electronic programmable...calculator in a first course in (college) calculus."

2. Rationale

Although studies have shown that the computer is an effective means of improving achievement in first-year calculus, the programmable mini-computer seems more appropriate in terms of calculating power needed, long-term cost, mobility of machine, and the lack of a programming language requirement.

3. Research Design and Procedure

Students in two intact sections of a first-year college calculus course were the subjects, with one section serving as the control and the other as the experimental group. Each group had 3 lectures plus a recitation session each week for a semester. Instruction for the experimental group included extra topics (more on limits of functions, applications of the derivative, and numerical integration) with additional exercises using the mini-computer. Six tests (limits of functions, continuity, interrelationships between a function and its first two derivatives, local extrema of a function, solving verbal problems with max/min and related rates, the definite integral) gave dependent measures. By blocking on "previously demonstrated achievement in mathematics" the experimenter used a 3 (levels of achievement) by 2 (treatments) design. The dependent measures were analyzed separately by ANOVAs (unequal cell frequencies, total n in the 30s).

4. Findings

Significant differences ($p < .05$) favoring the experimental group appeared on 3 of the 6 measures: interrelationships between a function and its first two derivatives, solving verbal problems with max/min and related rates, the definite integral. No other statistics were reported.

5. Interpretations

"The results of the study confirm the investigator's belief that the mini-computer can effectively be used to improve instruction in introductory

mathematics courses, and that its capability is comparable to that of a computer when used as a teaching aid in beginning calculus."

Critical Commentary

The primary strength of the study, as reported, is that it lasted for a semester. One could argue that, over a semester, interactions and events (other than the mini-computer) in classes as small as those in the study could make the classes far "different." But it is pleasing to read about a semester-long study rather than those one or two hour studies so common in the literature. Perhaps the brevity of the report is to blame, but little else good can be said for the study, as reported.

a. It is not reported whether the same instructor taught both classes. In view of the investigator's belief noted in the Interpretations above, one would hope that he taught neither section.

b. It is not reported to what extent the students actually used the mini-computer.

c. After "additional topics in limits of functions, applications of the derivative and numerical integration," it would seem that the experimental students should have had an advantage on 2 of the 3 tests for which significance was reported; it is perhaps surprising that there was not significance on the limits-of-functions test. Perhaps the additional topics--not the mini-computer--deserve the "credit" for the differences noted.

d. No mean scores are reported, so one cannot judge the practical significance of the differences. Nor can one subdue the surprise that there were apparently no significant differences among the 3 "prior achievement" levels.

e. Since the results on the 6 tests were probably correlated, a multivariate analysis would have been more appropriate--or to say it more precisely, slightly less inappropriate--than the 6 ANOVAs. While the lack of assured randomness can perhaps be overlooked on pragmatic grounds (most of us who have watched first-year students believe that they do register at random!), one cannot ignore the unequal cell sizes (no details), no reported concern for homogeneity of variances, and the small n.

f. No end-of-semester test results are reported. Such a test might have given valuable information on the longer-term effects of the mini-computer.

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AN EXPERIMENTAL STUDY OF THE EFFECTIVENESS OF A FORMAL VERSUS AN INFORMAL PRESENTATION OF A GENERAL HEURISTIC PROCESS ON PROBLEM SOLVING IN TENTH-GRADE GEOMETRY. Post, Thomas R. and Brennan, Michael L. Journal for Research in Mathematics Education, v7 n1, pp59-64, January 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by William B. Moody, The University of Delaware, Newark.

1. Purpose

The study compared the problem-solving ability of subjects taught a General Heuristic Problem-Solving Procedure with subjects not receiving any formal instruction in problem solving.

2. Rationale

Instructional materials were developed to be used with groups of subjects. These materials included group discussion activities concerning the nature of a problem, its location and definition, and the components of a problem-solving process. Problem tasks were included to be solved by groups of subjects. The materials were similar in nature to many aspects of Polya's approach to a heuristic process of problem solving. This study attempted to ascertain whether an extremely formal instructional approach would enhance success at problem solving.

3. Research Design and Procedure

The General Heuristic Problem-Solving Procedure was defined under the subheadings of: I. Recognition, Clarification, and Understanding of the Problem; II. Plan of Attack--Analysis; III. Productive Phase; and IV. Validating Phase-Checking-Proving. A final 20-item pretest-posttest was constructed from a sample of piloted problems. The pretest was administered to the ninth-grade students in the spring prior to conducting the study the following fall.

The subjects were 94 tenth-grade high school students enrolled in the top four classes of six possible geometry classes. All subjects had completed an algebra course. The subjects were rank-ordered by pretest scores, paired by matching scores, and then the pairs were randomly split into experimental and control groups. A median split was used to divide both treatment groups into high and low subgroup components. The experimental group received a teacher-directed, group-paced learning package based on the General Heuristic Problem-Solving Procedure. The control classes received no formal instruction in problem solving. The experimental model was a Pretest/Posttest Control Group Design as described by Campbell and Stanley (1963). A simple two-way analysis of variance was employed to compare posttest means.

4. Findings

The analysis produced no significant difference between the experimental and control group mean scores on the problem solving posttest. It was noted that the combined low-group posttest was significantly higher than the combined low-group pretest mean. This was not true for the combined high-group means.

5. Interpretations

Primarily, the conclusion was that formal instruction, employing a heuristic problem-solving attack, was not effective in improving the subjects' problem-solving ability. It was noted that this finding was consistent with that of a similar study conducted by one of the authors. It was suggested that the construction of valid instruments for measuring problem solving abilities is critical to the production of worthwhile research in this area.

Critical Commentary

The report of this investigation does little to shed light on the mysteries of problem-solving behavior. The reader has no indication of the type of problems employed as the criterion measure. There are no examples or descriptions of the problems. It is also not clear how the correctness of the problem solutions was determined. Was evidence gathered to substantiate whether the experimental or control subjects' behaviors differed when solving the posttest problems? Perhaps the experimental subjects employed pretest-type of attacks on the posttest items in spite of the experimental treatment. In other words, there is no indication that subjects in the experimental condition differed from control subjects by any problem-solving paradigm. It would seem that the actual method of attack would be of greater importance than the right-wrong scores on the problems. Great effort is given to develop a formal instructional package of materials and then it seems the subjects' rote responses to the problems are used as a measure of its effect.

Another glaring omission in this report was the time interval for the experimental treatment. Since there was no significant general gain from pretest to posttest scores for the population, it is hard to believe that much time was spent giving any type of problem-solving instruction or practice.

It is extremely difficult to draw any inferences from such sketchy reporting of research. Research in the area of problem solving must be structured in such a way as to examine the specific behavior of the individual's performance. Unless this report omitted procedures followed in the study, it does not seem that such a structure existed.

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EJ 118 366 TM 501 516

CHILDREN'S SOLUTION PROCESSES IN ARITHMETIC WORD PROBLEMS. Rosenthal, Daniel J. A.; Resnick, Lauren B. Journal of Educational Psychology, v66, pp817-825, December 1974.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Billie Earl Sparks, University of Wisconsin-Eau Claire.

1. Purpose

The purpose of this study was to investigate the effects that three variables have on third-grade children's achievement and solution time in solving verbal problems in mathematics. The three variables were: (1) the order of mention of chronological events, (2) the identity of the unknown set, and (3) the type of verb associated with the change set.

2. Rationale

Processes involved in the solution of verbal problems in mathematics are important to understand, since situations outside the mathematics classroom usually call for the construction of a problem representation as well as a solution. The identification of problem variables which affect achievement and solution time should suggest possibilities for the type of processing model used by the problem solver. The study derived from several studies by Suppes and associates, who postulate models by which numerical information is stored and transformed.

3. Research Design and Procedure

Two groups of third-grade pupils from two different public schools were used in two replications of the experiment. In Experiment 1, 29 pupils (15 boys, 14 girls) were involved, while 34 pupils (15 boys, 19 girls) were used for Experiment 2. The first experiment was done near the start of the third-grade year, while the second was conducted near the end of the third-grade year. The subjects were determined to be a representative cross-section of the pupil population with regard to academic standing.

The experimenters constructed a series of verbal problems which systematically varied with respect to the three variables: order of mention, identity of the unknown set, and type of verb. The variables may be described in line with the type of problem used. Each problem consisted of three clauses, one containing a "starting set," one a "change set," and one an "ending set." The order of mention variable concerns the chronological order of statement: starting, change, ending, or ending, change, starting. The identity of the unknown set variable involves whether the starting or ending set was the unknown set. The type of verb variable concerns whether eight item forms were derived from this 2x2x2 classification of problem variables. Four problems were generated meeting each item form criterion.

The subject was seated before a response box and a 0.01-second timer. When a problem was solved, the subject pressed a switch corresponding to the correct answer. This allowed recording of correctness of answer and reaction time.

A 2x2x2 analysis of variance with repeated measures on every factor was performed both on number of errors made and on reaction time for both experiments. For the reaction time analysis, only the times for correctly solved problems from three out of four items for each item form was analyzed. The first problem from each item form was eliminated to provide more stable times.

4. Findings

The following statistical results were determined for both experiments:

(1) The interactions of identity of unknown set with order of mention and identity of unknown set with type of verb were significant for the error data.

(2) The three main effects of identity of the unknown set, order of mention, and type of verb were significant for the error data. In Experiment 1 the effect for type of verb was only marginally significant ($p < 0.06$).

(3) The main effect of identity of unknown set was significant for the reaction time data.

5. Interpretations

The authors claim support for three of their hypotheses. These are: (1) backward order of mention problems produced more errors than did forward order of mention problems; and problems with an unknown starting set produced (2) more errors and (3) longer reaction times than did problems with ending set unknown. These statements are made within the limitations that almost no errors occured on ending set unknown problems and, because for reaction time only the times for correct answers were used, many subjects did not have a full complement of cell means and were excluded from the analysis. In fact only four subjects in Experiment 1 and twelve subjects in Experiment 2 were used in the reaction time analysis.

The findings are used to lend support to two models by which subjects possibly process numerical information. These are a canonical transformation model and a trial and error model.

Critical Commentary

Any interpretations drawn from the reaction time data must be very limited. With only three items per cell, if even one were missed the reaction time variance is very unstable. This, coupled with the small sample sizes, makes it unwise to generalize in this part of the study.

As the authors note, any interpretations of the error data must be made cognizant of the two significant interactions. The main effects of verb and order are attributed almost entirely to differences in the starting set unknown dimension. In fact, I feel the interpretations would have been more meaningful had the design been split along the identity of unknown set dimension after the interactions were found. For example, in Experiment 1 order of mention is a significant effect when all problems have the starting set unknown, but is not significant when all problems have ending set unknown.

The effect of placement of the unknown is a highly significant effect in this study and does seem to be consistently significant in several recent studies.

The postulated processing models certainly deserve more attention. The authors make no attempt to apply their findings, but it is certainly hoped that research such as this can soon be used in construction of curriculum materials and development of instructional strategies for the classroom. It is important to know whether the form of the question rather than lack of knowledge is causing errors.

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EJ 098 531 SE 511 192

FIVE FORMS OF WRITTEN FEEDBACK TO HOMEWORK IN A MATHEMATICS COURSE FOR ELEMENTARY TEACHERS. Schoen, Harold L. and Kreye, Bettibel C. Journal for Research in Mathematics Education, v5 n3, pp140-145, December 1971.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James W. Wilson, University of Georgia.

1. Purpose

The research reported in this article was an attempt to vary systematically the form of written feedback to student errors on mathematics homework assignments and determine the effect on student achievement, retention, and attitude.

2. Rationale

Previous research is cited on teacher feedback to homework assignments as an independent variable. Two variables, specificity of feedback (S) and personalization (P), were considered in establishing four types of feedback: 1) feedback specific to the error and utilizing the student's first name in responding; 2) feedback giving the correct answer and reasons why it is correct but not referring to the error made, and using the student's first name; 3) feedback specific to the error but not using the student's name; and 4) feedback giving the correct answer and reasons why it is correct, but not using the student's name.

3. Research Design and Procedure

The subjects were 147 prospective elementary school teachers enrolled in mathematics classes. Homework was required in the once-per-week laboratory sessions. The students were assigned randomly to one of the four treatment groups or to a control group. All were unaware of the experiment.

The dependent measures were 15-item achievement tests--a mid-term, a final, and a re-admission of the final for a retention measure--and an attitude measure.

4. Findings

The ANOVA found no significant differences on any comparison except one and the authors state it is doubtful that differences can be attributed to treatment.

5. Interpretations

The experimental treatments were too limited to cause any difference in achievement and the test instruments were very unreliable. Therefore, feedback variations do not appear to have any effect, but the present study

does not provide an adequate test. A refinement of the present study is planned.

Critical Commentary

The authors are to be commended for their candid recognition of the limitations of the study and their caution in presenting interpretations. Further, the writing is concise.

This article contributes very little. Its only value is for introducing research on varying homework feedback. The study is a pilot study with treatments too sparse to expect any results and with instruments too unreliable to measure even strong effects. The authors would have been better served to elaborate on the rationale and the significance of this line of research in a JRME Forum article, put aside this pilot study data, and repeat the study with more refined treatments and instruments.

Moreover, the article does not convince the reader of the educational significance of such research. If feedback is important, why are specificity and personalization the most critical variables? No reason is given. Frequency of feedback might be an alternative variable of more potential.

If the example items given in this article, such as

$$-3 - (-2) = (0,3) - (0,2) = (0,3) + (2,0) = (2,3) = -1$$

are representative of the mathematics content in the homework assignments, this writer believes that the selection of appropriate and interesting material might be an educationally significant variable. That is, an ordered-pair approach to the integers might reinforce negative attitudes toward mathematics and inhibit learning. It certainly needs something more powerful than once-per-week feedback on homework to motivate its acceptance by prospective elementary school teachers.

The research strategy is interesting because it is non-intrusive. That is, the treatments can be assigned and implemented with the individual as the sampling unit, without the subjects being aware of the experiment.

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EJ 088 480 SE 509 984

PUPIL PERFORMANCE ON EXAMPLES INVOLVING SELECTED VARIATIONS OF THE DISTRIBUTIVE IDEA. Weaver, J. Fred. Arithmetic Teacher, v20, pp697-704, December 1973.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald D. Brazier, Virginia Polytechnic Institute and State University.

1. Purpose

To investigate the relationship between pupils' performance on items involving the distributive principle and varying characteristics of those items.

2. Rationale

Investigations (Gray, 1965; Schell, 1968) have shown that children in the middle-elementary grades can learn some aspects of distributivity. However, this concept is more difficult to learn than others--commutativity, for example (Flournoy, 1964; Crawford, 1965). It is conjectured that pupils' sensitivity to distributivity may be related differentially to certain mathematical or other characteristics of examples which pupils encounter. Three such characteristics investigated were context, distributive form, and example format. Context was either regrouping sets (3 sets of 8 and 9 sets of 8) or multiplication-addition [(3 x 8) + (9 x 8)]. Distributive form was either left-distributive or right-distributive. Example format was either horizontal or vertical.

3. Research Design and Procedure

Eight 8-item tests were constructed so that the example characteristics were balanced in each test. Each test item involved whole numbers k, q, and p [as in (k x p) + (k x q)] subject to the following constraints:

For items 1, 2, 5 and 6 of each test,

$4 < k < 10$; $\quad 1 < p < 10$
$1 < q < 10$; $\quad p + q > 10$

For items 3, 4, 7 and 8 of each test,

$11 < k < 17$; $\quad 1 < p < 10$
$1 < q < 10$; $\quad p + q > 10$

The tests were given to 2000 pupils drawn from more than 100 fourth-, fifth-, sixth-, and seventh-grade classrooms in two different school districts. The tests were administered in the classrooms by a single person in each district (not the classroom teacher). Each pupil worked

with one of the randomly distributed tests under a 12-minute time-limit. The test booklets contained one item per page and the pupils were instructed not to turn back at any time to a previously completed, attempted, or omitted page.

The dependent variable was the number of criterion responses--that is, the response that would result if a pupil applied distributivity without any computational error. The data presented in the study were from only one of the districts, since the pattern of results was so similar for the two districts.

4. Findings

The performance levels were very low, ranging from 2% for the fourth-grade pupils to 11% for the seventh-grade pupils. At each grade level at least 90% of the pupils gave criterion responses on less than three of the eight test items. This unexpectedly low performance prompted marked changes in the anticipated analysis of data. At most, a detailed view of the data may suggest tendencies.

In Table 1, the data on the relationship between performance and item characteristics are presented. The percentages seem to indicate that pupils had a greater tendency to give criterion responses to regrouping-sets items than to multiplication-addition items and to right-distributive than to left-distributive items.

Table 1

	Item Characteristic	Percent of Criterion Responses			
		Grade 4	Grade 5	Grade 6	Grade 7
A1.	Regrouping-sets context	2.3	5.1	8.5	15.6
A2.	Multiplication-addition context	1.1	1.7	2.2	6.0
B1.	Left-distributive form	0.8	1.9	2.7	5.0
B2.	Right-distributive form	2.6	4.9	8.0	16.6
C1.	Horizontal example format	1.9	3.4	5.4	11.0
C2.	Vertical example format	1.5	3.4	5.4	10.6

Since it is not inconceivable that pupil performance may be sensitive to certain combinations of item characteristics in an interacting, differential manner, data relevant to such an interaction are presented in Table 2. It seems that distributive form is more markedly related to pupil performance within the regrouping-sets context than within the multiplication-addition context.

Table 2

/n Item Characteristic Interaction

Item Characteristic	Percent of Criterion Responses			
	Grade 4	Grade 5	Grade 6	Grade 7
A1. Regrouping-sets context				
B1. Left-distributive form	0.7	2.1	3.6	5.0
B2. Right-distributive form	3.8	8.0	13.4	26.3
A2. Multiplication-addition context				
B1. Left-distributive form	0.9	1.7	1.8	5.0
B2. Right-distributive form	1.4	1.8	2.7	6.9

Student performance was further analyzed by investigating the pattern of errors. Aside from purely computational errors, two types of conceptual errors were found: Category I would be exemplified by an answer such as 24 x 72 to the item (3 x 8) + (9 x 8); Category II would be exemplified by an answer such as 12 x 16 to the same item. Table 3 summarizes the relative frequencies of the types of errors in relation to selected item characteristics. The percentages seem to indicate Category I and Category II responses are related to context and format.

Table 3

Two Categories of Conceptually Incorrect Responses

Item Characteristic	Percent of Responses			
	Grade 4	Grade 5	Grade 6	Grade 7
Category I Responses				
A1. Regrouping-sets context	5.4	14.2	15.9	9.3
A2. Multiplication-addition context	14.8	31.4	38.5	39.8
C1. Horizontal example format	11.0	25.1	32.7	29.0
C2. Vertical example format	9.1	20.5	21.7	20.0
Category II Responses				
A1. Regrouping-sets context	19.3	16.0	26.5	35.8
A2. Multiplication-addition context	10.0	7.6	14.1	18.0
C1. Horizontal example format	6.0	5.0	11.0	17.7
C2. Vertical example format	23.3	18.6	29.6	36.2

5. Interpretations

Though only suggestive, the data indicate that children's comprehension of and sensitivity to distributivity is not independent of factors such as context, form and format. Such factors may obscure the underlying principle of distributivity and hinder pupils' generalizing across specific differences. The nontrivial relative frequency of Category I and Category II responses and their relation to attributes such as context and format raise the issue of how instruction can be improved so as to prevent the establishment of misconceptions.

Critical Commentary

The low level of the subjects' performance in this study prevented a standard statistical analysis of the data. This was unfortunate because the indications of the data are very interesting. The interaction of context and form characteristics which indicates that the right-distributive form in a regrouping-sets context is appreciably easier than any other combination is not totally surprising. However, by the time students reach seventh-grade it would be expected that they would be more capable of generalizing distributivity across different modes of representation than this study suggests. Likewise, the data on types of conceptual errors are suggestive of a lack of generalization by the pupils.

There is very little to criticize about the study. The sample could have been more adequately characterized--the reader must assume that the subjects were average elementary school students studying a standard mathematical curriculum. Also, the background of the interaction of characteristics with performance is not developed at all. Are we to assume that no previous research into interactions between item characteristics and sensitivity to mathematical concepts has been conducted? Possibly the audience for which the journal is intended was being spared a highly technical report. There are other stylistic quirks of presentation - welcome ones, I hasten to add - that make the report of this study more informal and readable than many others.

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ED 115 490 SE 019 853

STUDY I--PRELIMINARY DEVELOPMENT OF AN ORAL SPEED TEST FOR ARITHMETIC FACTS. Wiles, Clyde A.; Romberg, Thomas A. In Romberg, Thomas A. (ed.), Four Studies on Aspects of Assessing Computational Performance. Technical Report No. 297. Wisconsin Research and Development Center for Cognitive Learning. The University of Wisconsin-Madison, July 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jack D. Wilkinson, University of Northern Iowa.

1. Purpose

a. To develop an initial prototype of a valid and reliable measure of students' speed at recalling addition facts.

b. To examine the relationship between students' rate of response on basic arithmetic combinations (facts) and their learning of algorithms that use those basic combinations.

2. Rationale

Teachers generally believe that rapid response rates by students on the basic facts are prerequisite to their learning of the algorithms that employ the facts. The proliferation of "drill and practice" programs for use in the elementary school supports the contention that such response rates are highly valued in instructional outcomes.

3. Research Design and Procedure

Earlier investigation (Wiles, 1973) had revealed algorithmic interference when facts were presented in a visual form. Examples of algorithmic interference are:

$$\begin{array}{r} {}^{1}6 \\ +8 \\ \hline 14 \end{array} \quad \text{and} \quad \begin{array}{r} {}^{1} \\ 15 - 9 = 6 \end{array}$$

To lessen the potential for algorithmic interference, this investigation used a series of three oral tapes. The tapes presented sets of addition facts. Children were also asked to work two-digit addition problems to see if they had mastered an algorithm. Addition facts and the addition algorithm were chosen because:

(1) Children who do not know an addition algorithm probably do not know any other computational algorithms.

(2) It should be possible to find children from a broad ability range who vary widely in their rates of response to the basic facts and who do not know an adequate algorithm for adding a pair of two-digit numbers.

Tape 1 contained 7 sets of 10 addition facts read at different time intervals. Each set of 10 facts consisted of 5 easy items and 5 difficult items. Easy items consisted of 2 one-digit numbers (neither zero) where the sum is greater than 2 but less than 10. Difficult facts were those where the sum was 10 or greater. The time intervals for the 7 sets of facts varied from 15 seconds to 4 seconds. The fact items were also subject to constraints dealing with commuted forms, doubles (4 + 4), sums of 10, and items using 9 as one addend.

Tape 1 was administered to two groups of third graders and one group of second graders in the fall of 1972. All of the subjects were from a suburban school district in central Wisconsin. One of the third-grade groups was a selected group of 7 children identified by their teacher as of average or less than average mathematical ability. The other third-grade group of 6 children was identified by the same teacher as being of high mathematical ability. The second-grade children were a random selection of 10 children from an intact second-grade class.

After the tapes were presented, the students were asked to find the sum of 2 two-digit numbers. Third graders were asked to complete 10 problems, 5 without regrouping and 5 with regrouping. Second graders were asked to complete 2 problems, 1 without regrouping and 1 with regrouping.

In light of the experiences with Tape 1, two additional tapes were prepared. Three written test booklets were also designed. These materials were not piloted as a part of this study.

4. Findings

A major objective of an adequate facts speed test is that it separates those subjects who are responding from memory from those who must count to find the solution. It was decided that the test provided by Tape 1 was inadequate in this regard.

The 7 sets of basic fact items had time intervals between fact readings of 15 sec., 15 sec., 10 sec., 8 sec., 6 sec., 4 sec., and 10 sec. Generally, these intervals were found to be too long. Students demonstrated a loss of task focus. The time intervals were longer than the children required to determine their response. In the revision of Tape 1, the number of blocks of 10 items was reduced from 7 to 5 and the time intervals were 8 sec., 8 sec., 5 sec., 3 sec., and 12 sec. The last block was intended for use as a measure of facts mastery. It was decided that the shorter time intervals told as much about mastery of the facts (in terms of the ability to produce sums by counting, if necessary) as did the longer time intervals.

In the pilot of Tape 1, it was common to observe a child who was "counting up" to find the sum. When difficult facts were presented, the child may not have finished before the next item was read; the child would then do the next one if it was easy, and go back to finish or recount the difficult item.

The revised tape was modified so that in each block of 10 items, all five of the easy items were read first.

5. Interpretations

This study grew out of a need for an instrument to measure students' speed at recalling addition facts. This had seemed to be a very easy task, but it proved to be much more difficult than anticipated.

The obvious next step is the piloting of the revised tapes and the booklets which accompany the tapes. It would be anticipated that a selection of one of these forms could then be made, and reliability estimates obtained for the selected test.

Critical Commentary

The study displays much care in the selection of criteria for facts and the presentation of the data. The author's observation that Tape 1 was not able to accomplish its objectives is a contribution. It is unfortunate that they were not able to report on findings from the revised tapes.

The selection of third-grade subjects was made by teacher judgment. The danger of misclassification by a teacher is real and conclusions must be considered in that light.

The relation of basic fact skills to computational success was not clear. This type of information can be useful to teachers who adhere to the assumption that mastery of basic facts is a necessary prerequisite to the development of an algorithm. This study is a small step in providing teachers with needed information regarding the role of fact mastery as it relates to algorithm development.

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